Proc. of Int. Conf. on Emerging Trends in Engineering & Technology, IETET

Mode Shape Analysis of Free-Free Beam

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***Abstract*—Vibration effects include unwanted behaviour such as improper working, early failure, noise problems etc. System performance decreases due to vibrations. Therefore, we need to understand the vibration pattern of structures, which is a function of natural frequencies as well as mode-shapes. Such understanding is must for developing a good design. Mode-shapes helps us in understanding the way in which is structure is going to vibrate when put under free or forced vibrations. This paper presents a basic work related to development of Matlab program which can be useful for analysing the natural frequencies and mode-shapes of a free-free beam. Extreme as well as a number of intermediate positions of mode shapes of the free-free beam structure have been plotted. The work is supposed to be helpful for beginner researchers to understand the mode-shape analysis of beams.**

***Index Terms*— Mode shape analysis, Finite element method, Free-free beam, Matlab.**

I. INTRODUCTION

In today's world there is a wide use of beams especially in the field of mechanical and structural engineering [1]. Beam is a structural element which is strong enough to resist against bending and is commonly used in buildings, rooftops, bridges etc.. The beam designing basically depends on their shape, material, dimensional requirements, and most importantly the vibration behaviour [2]. The material properties of the beam mainly include its modulus of elasticity and mass density. These material properties affect the vibration behaviour of the beam, because the mass matrix is a function of mass density and the stiffness matrix is a function of the modulus of elasticity. Beam analysis also requires the use of mechanical engineering design principles. The aim of this analysis is to understand the structural dynamic behaviour of the beam such as natural frequencies, eigenvalues and mode shape analysis. Such an analysis is also called as modal analysis [3]. The basic use of modal analysis is to analyse, determine and measure the dynamic behaviour of structures [4]. Modal analysis is also helpful in model updating problems [5], [6], [7], [8] as well as in damage detection of structures [9]. The modal analysis has discovered varied applications in the field of mechanical, acoustical instruments, aeronautical, civil, transportation, nuclear plants and space structures. The demands for highly efficient machines are increasing day by day. The performance of machines depends upon its design and the type of material used. Lighter materials are used now-a-days for construction of machines. It further results in the increased vibration and noise pollution. Excessive vibration can also result in increasing the sensitivity of machines to high frequency of noise and pre-mature failure. Excessive vibration produced by the machines also affect the normal life of workers by causing nerve damage, back pain etc. Thus the system will vibrate at

Grenze ID: 02.IETET.2016.5.41

one or more of its natural frequencies, which are properties of the system dynamics, due to its stiffness and mass distribution.

This paper presents a mode-shape analysis of free-free beam. Computer programs have been prepared in Matlab platform. These programs have been used to study and analyze the natural frequencies and mode-shapes of a free-free beam. First few modes of the free-free beam have also been drawn to represent the modal behaviour of the beam in a graphical format.

II. Mode Shape Analysis of Beam

Mode shape analysis is the method in which the models of the system are obtained using finite element method [10]. In mode shape analysis, a complex structure is discretized into simpler shapes. These simpler shapes are called as finite elements. In this work, the dimensions of the free-free beam structure for analysis are 1000 x 50 x 5 mm, density is 7800 kg/m^3. The FE model of the beam contains a number of one-dimensional finite elements having a node at each end. In this analysis, the all nodes have two degree of freedom, one represent displacement in y-direction and the other shows rotation in z-direction. Each finite element of the structure is expressed as elemental mass and stiffness matrix. Mass matrix is represented by (1) and stiffness matrix has been shown in (2), where, m_e, k_e, ρ, A, a, E, I are mass matrix, stiffness matrix, mass-density, area of cross-section, half-length, modulus of elasticity and rectangular moment of inertia of cross-section of beam element respectively.

$$[m_e] = \frac{\rho A a}{105}\begin{bmatrix} 78 & 22a & 27 & -13 \\ 22a & 8a^2 & 13a & -6a^2 \\ 27 & 13a & 78 & -22a \\ -13a & -6a^2 & -22a & 8a^2 \end{bmatrix} \quad (1)$$

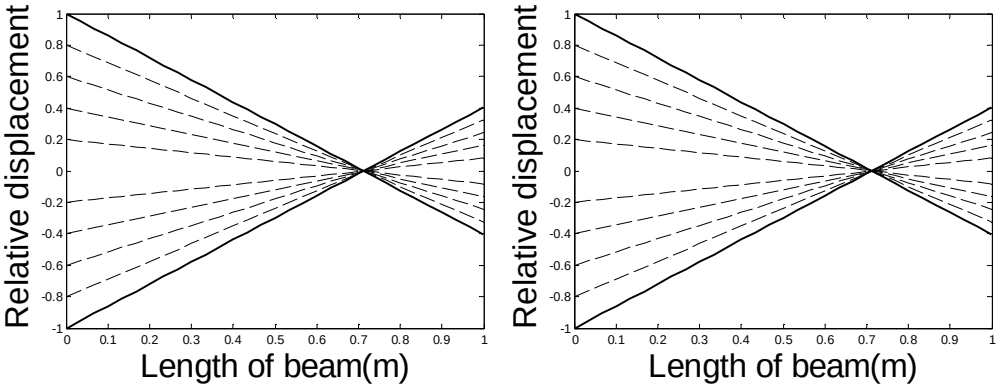
$$[k_e] = \frac{EI}{2a^3}\begin{bmatrix} 3 & 3a & -3 & 3a \\ 3a & 4a^2 & -3a & 2a^2 \\ -3 & -3a & 3 & -3a \\ 3a & 2a^2 & -3a & 4a^2 \end{bmatrix} \quad (2)$$

These matrices are then assembled to develop to global mass matrix and stiffness matrix respectively. Global matrices are then executed in Matlab to evaluate the natural frequencies as well as mode-shapes of the beam under consideration. It is seen that first two frequencies come out to be zero, which justifies the presence of two rigid body modes. Table 1 shows the natural frequencies of first six vibration modes.

TABLE I. Natural Frequencies of First Six Modes

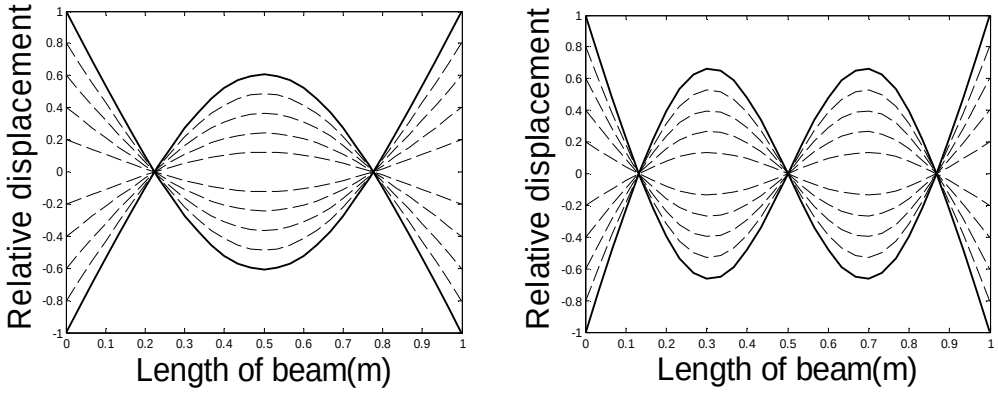
Mode Number	Natural Frequency (Hz)
1	9.8
2	27.0
3	53.0
4	87.6
5	130.9
6	182.8

Fig. 1 shows the mode shape for rigid body modes of the beam. Mode-shapes of first to six modes of vibration have been presented in Fig. 2. The solid lines show the extreme positions while the dashed lines represent the intermediate positions of beam locations during corresponding mode-shape.

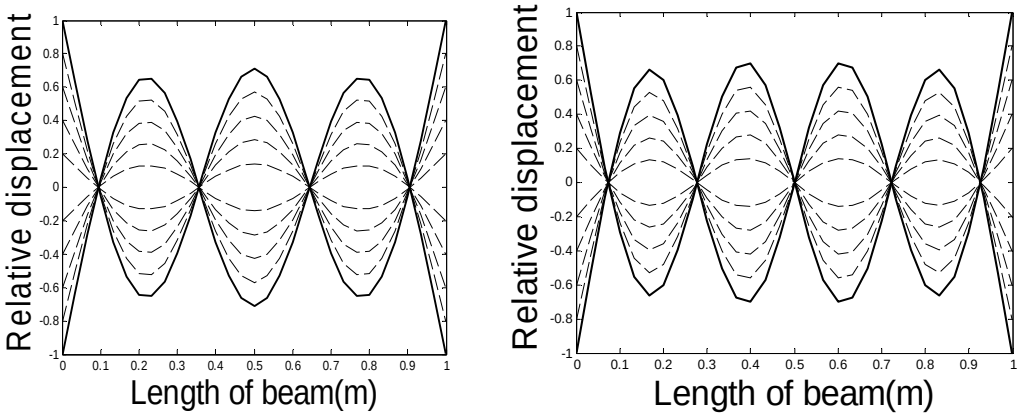


(a) (b)

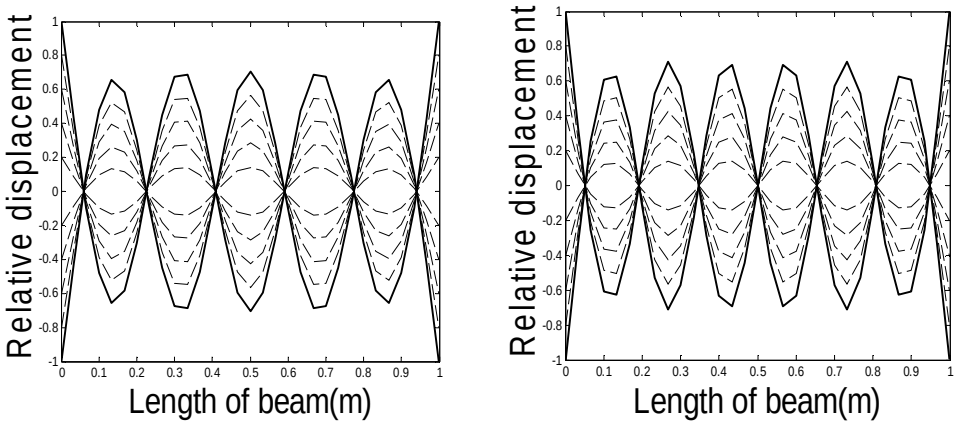
Figure 1. Rigid body modes of a free-free beam



(a) (b)



(c) (d)



(e) (f)

Figure 1. Mode-shapes of first six modes (a) first mode (b) second mode (c) third mode (d) fourth mode (e) fifth mode (f) sixth mode

III. Conclusions

The Free-Free beam structure has been analysed for mode shape analysis, eigenvalues and mode shapes. FE model help us to successfully determine and analyse these dynamic behaviour. The analysis is helpful in FE model updating, designing of structures also for structural modification. The work has been executed in Matlab.

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